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Local stability of the Pexiderized Cauchy and Jensen's equations in fuzzy spaces

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Abstract

Lex X be a normed space and Y be a Banach fuzzy space. Let $D = \{(x, y) \in X \times X : \|x\| + \|y\| \geq d\}$ where $d > 0$. We prove that the Pexiderized Jensen functional equation is stable in the fuzzy norm for functions defined on D and taking values in Y. We consider also the Pexiderized Cauchy functional equation.

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1. Introduction

The functional equation (ξ) is *stable* if any function g satisfying the equation (ξ) approximately is near to the true solution of (ξ).

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms:

Let G_1 be a group and let G_2 be a metric group with the metric $d(\cdot,\cdot)$. Given $\varepsilon > 0$, does there exist $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \varepsilon$ for all $x \in G_1$?

In other words, we are looking for situations when the homomorphisms are stable, i.e., if a mapping is almost a homomorphism, then there exists a true homomorphism near it. If we turn our attention to the case of functional equations, then we can ask the question: When the solutions of an equation differing slightly from a given one must be close to the true solution of the given equation.

In 1941, Hyers [2] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1950, Aoki [3] provided a generalization of the Hyers' theorem for additive mappings, and in 1978, Th.M. Rassias [4] succeeded in extending the result of Hyers for linear mappings by allowing the Cauchy difference to be unbounded (see also [5]). The stability phenomenon that was introduced and proved by Th.M. Rassias is called the *generalized Hyers-Ulam stability*. Forti [6] and Găvruta [7] have generalized the result of Th.M. Rassias, which permitted the Cauchy difference to become arbitrary unbounded. The stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem. A large list of references can be found, for example, in [8-29].

Following [30], we give the following notion of a fuzzy norm.

Definition 1.1. [30] Let X be a real vector space. A function $N : X \times \mathbb{R} \to [0, 1]$ is called a *fuzzy norm* on X if, for all $x, y \in X$ and $s, t \in \mathbb{R}$,

(N_1) $N(x, t) = 0$ for all $t \leq 0$;
(N_2) $x = 0$ if and only if $N(x, t) = 1$ for all $t > 0$;
(N_3) $N(cx, t) = N(x, \frac{t}{|c|})$ if $c \neq 0$;
(N_4) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$;
(N_5) $N(x,\cdot)$ is a nondecreasing function on $\mathbb{R}$ and $\lim_{t\to\infty} N(x, t) = 1$;
(N_6) for $x \neq 0$, $N(x,\cdot)$ is continuous on $\mathbb{R}$.

The pair (X, N) is called a *fuzzy normed vector space*.

Example 1.2. Let $(X, ||\cdot||)$ be a normed linear space and let $\alpha, \beta > 0$. Then,

$$N(x,t) = \begin{cases} \dfrac{\alpha t}{\alpha t + \beta\|x\|}, & t > 0,\ x \in X, \\ 0, & t \leq 0,\ x \in X \end{cases}$$

is a fuzzy norm on X.

Example 1.3. Let $(X, ||\cdot||)$ be a normed linear space and let $\beta > \alpha > 0$. Then,

$$N(x,t) = \begin{cases} 0, & t \leq \alpha\|x\|, \\ \dfrac{t}{t + (\beta - \alpha)\|x\|}, & \alpha\|x\| < t \leq \beta\|x\|; \\ 1, & t > \beta\|x\| \end{cases}$$

is a fuzzy norm on X.

Definition 1.4. Let (X, N) be a fuzzy normed space. A sequence $\{x_n\}$ in X is said to *be convergent* if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case, x is called the *limit* of the sequence $\{x_n\}$, and we denote it by $N - \lim x_n = x$.

The limit of the convergent sequence $\{x_n\}$ in (X, N) is unique. Since if $N - \lim x_n = x$ and N-$\lim x_n = y$ for some $x, y \in X$, it follows from (N_4) that

$$N(x-y,t) \geq \min\left\{N\left(x - x_n, \frac{t}{2}\right), N\left(x_n - y, \frac{t}{2}\right)\right\}$$

for all $t > 0$ and $n \in \mathbb{N}$. So, $N(x - y, t) = 1$ for all $t > 0$. Hence, (N_2) implies that $x = y$.

Definition 1.5. Let (X, N) be a fuzzy normed space. A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for any $\varepsilon > 0$ and $t > 0$, there exists $M \in \mathbb{N}$ such that, for all $n \geq M$ and $p > 0$,

$$N(x_{n+p} - x_n, t) > 1 - \varepsilon.$$

It follows from (N_4) that every convergent sequence in a fuzzy normed space is a Cauchy sequence. If, in a fuzzy normed space, every Cauchy sequence is convergent, then the fuzzy norm is said to be *complete*, and the fuzzy normed space is called a *fuzzy Banach space*.

Example 1.6. [21] Let $N : \mathbb{R} \times \mathbb{R} \to [0, 1]$ be a fuzzy norm on $\mathbb{R}$ defined by

$$N(x, t) = \begin{cases} \dfrac{t}{t + |x|}, & t > 0, \\ 0, & t \leq 0. \end{cases}$$

Then, $(\mathbb{R}, N)$ is a fuzzy Banach space.

Recently, several various fuzzy stability results concerning a Cauchy sequence, Jensen and quadratic functional equations were investigated in [17-20].

2. A local Hyers-Ulam stability of Jensen's equation

In 1998, Jung [16] investigated the Hyers-Ulam stability for Jensen's equation on a restricted domain. In this section, we prove a local Hyers-Ulam stability of the Pexiderized Jensen functional equation in fuzzy normed spaces.

Theorem 2.1. *Let X be a normed space, (Y, N) be a fuzzy Banach space, and $f, g, h : X \to Y$ be mappings with $f(0) = 0$. Suppose that $\delta > 0$ is a positive real number, and z_0 is a fixed vector of a fuzzy normed space (Z, N') such that*

$$N\left(2f\left(\frac{x+y}{2}\right) - g(x) - h(y), t + s\right) \geq \min\{N'(\delta z_0, t), N'(\delta z_0, s)\} \tag{2.1}$$

for all $x, y \in X$ with $||x|| + ||y|| \geq d$ and positive real numbers t, s. Then, there exists a unique additive mapping $T : X \to Y$ such that

$$N(f(x) - T(x), t) \geq N'(40\delta z_0, t), \tag{2.2}$$

$$N(T(x) - g(x) + g(0), t) \geq N'(30\delta z_0, t), \tag{2.3}$$

$$N(T(x) - h(x) + h(0), t) \geq N'(30\delta z_0, t) \tag{2.4}$$

for all $x \in X$ and $t > 0$.

Proof. Suppose that $||x|| + ||y|| < d$ holds. If $||x|| + ||y|| = 0$, let $z \in X$ with $||z|| = d$. Otherwise,

$$z := \begin{cases} (d + \|x\|)\dfrac{x}{\|x\|}, & \text{if } \|x\| \geq \|y\|, \\[2ex] (d + \|y\|)\dfrac{y}{\|y\|}, & \text{if } \|x\| < \|y\|. \end{cases}$$

It is easy to verify that

$$\begin{aligned} &\|x - z\| + \|y + z\| \geq d, \quad \|2z\| + \|x - z\| \geq d, \quad \|y\| + \|2z\| \geq d, \\ &\|y + z\| + \|z\| \geq d, \quad \|x\| + \|z\| \geq d. \end{aligned} \tag{2.5}$$

It follows from (N_4), (2.1) and (2.5) that

$$\begin{aligned}
&N\Big(2f\left(\frac{x+y}{2}\right)-g(x)-h(y),t+s\Big)\\
&\quad\geq \min\Big\{N\Big(2f\left(\frac{x+y}{2}\right)-g(y+z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{x+z}{2}\right)-g(2z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{y+2z}{2}\right)-g(2z)-h(y),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{y+2z}{2}\right)-g(y+z)-h(z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(2f\left(\frac{x+z}{2}\right)-g(x)-h(z),\frac{t+s}{5}\Big)\Big\}\\
&\quad\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned}$$

for all $x, y \in X$ with $||x|| + ||y|| < d$ and positive real numbers t, s. Hence, we have

$$N\Big(2f\left(\frac{x+y}{2}\right)-g(x)-h(y),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}\tag{2.6}$$

for all $x, y \in X$ and positive real numbers t, s. Letting $x = 0$ ($y = 0$) in (2.6), we get

$$\begin{aligned}
&N\Big(2f\left(\frac{y}{2}\right)-g(0)-h(y),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\},\\
&N\Big(2f\left(\frac{x}{2}\right)-g(x)-h(0),t+s\Big)\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned}\tag{2.7}$$

for all $x, y \in X$ and positive real numbers t, s. It follows from (2.6) and (2.7) that

$$\begin{aligned}
&N\Big(2f\left(\frac{x+y}{2}\right)-2f\left(\frac{x}{2}\right)-2f\left(\frac{y}{2}\right),t+s\Big)\\
&\quad\geq \min\Big\{N\Big(2f\left(\frac{x+y}{2}\right)-g(x)-h(y),\frac{t+s}{4}\Big),\\
&\qquad N\Big(2f\left(\frac{x}{2}\right)-g(x)-h(0),\frac{t+s}{4}\Big),\\
&\qquad N\Big(2f\left(\frac{y}{2}\right)-g(0)-h(y),\frac{t+s}{4}\Big),N\Big(g(0)+h(0),\frac{t+s}{4}\Big\}\\
&\quad\geq \min\{N'(20\delta z_0,t),N'(20\delta z_0,s)\}
\end{aligned}$$

for all $x, y \in X$ and positive real numbers t, s. Hence,

$$N\big(f(x+y)-f(x)-f(y),t+s\big)\geq \min\{N'(10\delta z_0,t),N'(10\delta z_0,s)\}\tag{2.8}$$

for all $x, y \in X$ and positive real numbers t, s. Letting $y = x$ and $t = s$ in (2.8), we infer that

$$N\Big(\frac{f(2x)}{2}-f(x),t\Big)\geq N'(10\delta z_0,t)\tag{2.9}$$

for all $x \in X$ and positive real number t. replacing x by $2^n x$ in (2.9), we get

$$N\left(\frac{f(2^{n+1}x)}{2^{n+1}}-\frac{f(2^n x)}{2^n},\frac{t}{2^n}\right)\geq N'(10\delta z_0,t)\tag{2.10}$$

for all $x \in X$, $n \geq 0$ and positive real number t. It follows from (2.10) that

$$\begin{aligned} N\Big(\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m}, \sum_{k=m}^{n-1} \frac{t}{2^k}\Big) &\geq \min \bigcup_{k=m}^{n-1} \Big\{ N\Big(\frac{f(2^{k+1}x)}{2^{k+1}} - \frac{f(2^k x)}{2^k}, \frac{t}{2^k}\Big) \\ &\geq N'(10\delta z_0, t) \end{aligned} \tag{2.11}$$

for all $x \in X$, $t > 0$ and integers $n \geq m \geq 0$. For any s, $\varepsilon > 0$, there exist an integer $l > 0$ and $t_0 > 0$ such that $N'(10\delta z_0, t_0) > 1 - \varepsilon$ and $\sum_{k=m}^{n-1} \frac{t_0}{2^k} > s$ for all $n \geq m \geq l$. Hence, it follows from (2.11) that

$$N\Big(\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m}, s\Big) > 1 - \varepsilon$$

for all $n \geq m \geq l$. So $\{\frac{f(2^n x)}{2^n}\}$ is a Cauchy sequence in Y for all $x \in X$. Since (Y, N) is complete, $\{\frac{f(2^n x)}{2^n}\}$ converges to a point $T(x) \in Y$. Thus, we can define a mapping $T : X \to Y$ by $T(x) := N - \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$. Moreover, if we put $m = 0$ in (2.11), then we observe that

$$N\Big(\frac{f(2^n x)}{2^n} - f(x), \sum_{k=0}^{n-1} \frac{t}{2^k}\Big) \geq N'(10\delta z_0, t).$$

Therefore, it follows that

$$N\Big(\frac{f(2^n x)}{2^n} - f(x), t\Big) \geq N'\Big(10\delta z_0, \frac{t}{\sum_{k=0}^{n-1} 2^{-k}}\Big) \tag{2.12}$$

for all $x \in X$ and positive real number t.

Next, we show that T is additive. Let $x, y \in X$ and $t > 0$. Then, we have

$$\begin{aligned} &N\big(T(x+y) - T(x) - T(y), t\big) \\ &\quad \geq \min \Big\{ N'\Big(T(x+y) - \frac{f(2^n(x+y))}{2^n}, \frac{t}{4}\Big), \\ &\qquad N'\Big(\frac{f(2^n x)}{2^n} - T(x), \frac{t}{4}\Big), N'\Big(\frac{f(2^n y)}{2^n} - T(y), \frac{t}{4}\Big), \\ &\qquad N'\Big(\frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n}, \frac{t}{4}\Big)\Big\}. \end{aligned} \tag{2.13}$$

Since, by (2.8),

$$N'\Big(\frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n}, \frac{t}{4}\Big) \geq N'(40\delta z_0, 2^n t),$$

we get

$$\lim_{n\to\infty} N'\Big(\frac{f(2^n(x+y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n}, \frac{t}{4}\Big) = 1.$$

By the definition of T, the first three terms on the right hand side of the inequality (2.13) tend to 1 as $n \to \infty$. Therefore, by tending $n \to \infty$ in (2.13), we observe that T is additive.

Next, we approximate the difference between f and T in a fuzzy sense. For all $x \in X$ and $t > 0$, we have

$$N(T(x) - f(x), t) \geq \min\left\{N\Big(T(x) - \frac{f(2^n x)}{2^n}, \frac{t}{2}\Big), N\Big(\frac{f(2^n x)}{2^n} - f(x), \frac{t}{2}\Big)\right\}.$$

Since $T(x) := N - \lim_{n\to\infty} \dfrac{f(2^n x)}{2^n}$, letting $n \to \infty$ in the above inequality and using (N) and (2.12), we get (2.2). It follows from the additivity of T and (2.7) that

$$\begin{aligned} N(T(x) - g(x) + g(0), t) &\geq \min\Big\{N\Big(2T\big(\frac{x}{2}\big) - 2f\big(\frac{x}{2}\big), \frac{t}{3}\Big), \\ &\qquad N\Big(2f\big(\frac{x}{2}\big) - g(x) - h(0), \frac{t}{3}\Big), \\ &\qquad N\Big(g(0) + h(0), \frac{t}{3}\Big)\Big\} \\ &\geq N'(30\delta z_0, t) \end{aligned}$$

for all $x \in X$ and $t > 0$. So, we get (2.3). Similarly, we can obtain (2.4).

To prove the uniqueness of T, let $S : X \to Y$ be another additive mapping satisfying the required inequalities. Then, for any $x \in X$ and $t > 0$, we have

$$\begin{aligned} N(T(x) - S(x), t) &\geq \min\Big\{N\Big(T(x) - f(x), \frac{t}{2}\Big), N\Big(f(x) - S(x), \frac{t}{2}\Big) \\ &\geq N'(80\delta z_0, t). \end{aligned}$$

Therefore, by the additivity of T and S, it follows that

$$N(T(x) - S(x), t) = N(T(nx) - S(nx), nt) \geq N'(80\delta z_0, nt)$$

for all $x \in X$, $t > 0$ and $n \geq 1$. Hence, the right hand side of the above inequality tends to 1 as $n \to \infty$. Therefore, $T(x) = S(x)$ for all $x \in X$. This completes the proof. □

The following is a local Hyers-Ulam stability of the Pexiderized Cauchy functional equation in fuzzy normed spaces.

Theorem 2.2. *Let X be a normed space, (Y, N) be a fuzzy Banach space, and $f, g, h : X \to Y$ be mappings with $f(0) = 0$. Suppose that $\delta > 0$ is a positive real number, and z_0 is a fixed vector of a fuzzy normed space (Z, N') such that*

$$N(f(x + y) - g(x) - h(y), t + s) \geq \min\{N'(\delta z_0, t), N'(\delta z_0, s)\} \tag{2.14}$$

for all $x, y \in X$ with $||x|| + ||y|| \geq d$ and positive real numbers t, s. Then, there exists a unique additive mapping $T : X \to Y$ such that

$$\begin{aligned} N(f(x) - T(x), t) &\geq N'(80\delta z_0, t), \\ N(T(x) - g(x) + g(0), t) &\geq N'(60\delta z_0, t), \\ N(T(x) - h(x) + h(0), t) &\geq N'(60\delta z_0, t) \end{aligned}$$

for all $x \in X$ and $t > 0$.

Proof. For the case $||x|| + ||y|| < d$, let z be an element of X which is defined in the proof of Theorem 2.1. It follows from (N_4), (2.5) and (2.14) that

$$
\begin{aligned}
&N(f(x+y)-g(x)-h(y),t+s)\\
&\quad\geq \min\Big\{N\Big(f(x+y)-g(y+z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(x+z)-g(2z)-h(x-z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(y+2z)-g(2z)-h(y),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(y+2z)-g(y+z)-h(z),\frac{t+s}{5}\Big),\\
&\qquad N\Big(f(x+z)-g(x)-h(z),\frac{t+s}{5}\Big)\Big\}\\
&\quad\geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned}
$$

for all $x, y \in X$ with $||x|| + ||y|| < d$ and positive real numbers t, s. Hence, we have

$$N\Big(f(x+y)-g(x)-h(y),t+s\Big) \geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\} \tag{2.15}$$

for all $x, y \in X$ and positive real numbers t, s. Letting $x = 0$ ($y = 0$) in (2.15), we get

$$
\begin{aligned}
&N(f(y)-g(0)-h(y),t+s) \geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\},\\
&N(f(x)-g(x)-h(0),t+s) \geq \min\{N'(5\delta z_0,t),N'(5\delta z_0,s)\}
\end{aligned} \tag{2.16}
$$

for all $x, y \in X$ and positive real numbers t, s. It follows from (2.15) and (2.16) that

$$
\begin{aligned}
&N(f(x+y)-f(x)-f(y),t+s)\\
&\quad\geq \min\Big\{N\Big(f(x+y)-g(x)-h(y),\frac{t+s}{4}\Big),\\
&\qquad N\Big(f(x)-g(x)-h(0),\frac{t+s}{4}\Big),\\
&\qquad N\Big(f(y)-g(0)-h(y),\frac{t+s}{4}\Big),\\
&\qquad N(g(0)+h(0),\frac{t+s}{4})\Big\}\\
&\quad\geq \min\{N'(20\delta z_0,t),N'(20\delta z_0,s)\}
\end{aligned}
$$

for all $x, y \in X$ and positive real numbers t, s. The rest of the proof is similar to the proof of Theorem 2.1, and we omit the details. □

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Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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